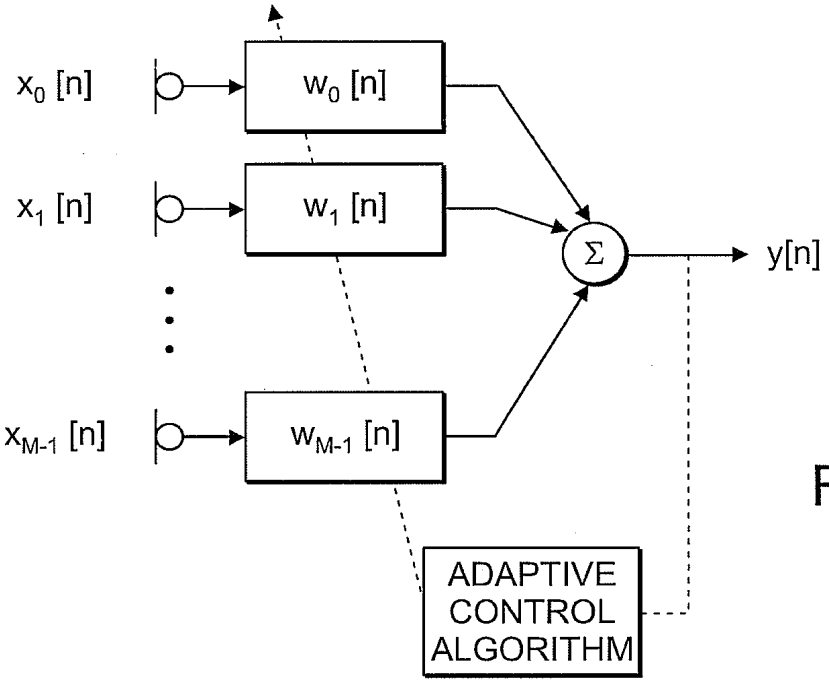

x_0 [n]
w_0 [n]
x_1 [n]
w_1 [n]
Σ
y[n]
x_{M-1} [n]
w_{M-1} [n]
ADAPTIVE CONTROL ALGORITHM

FIG. 1

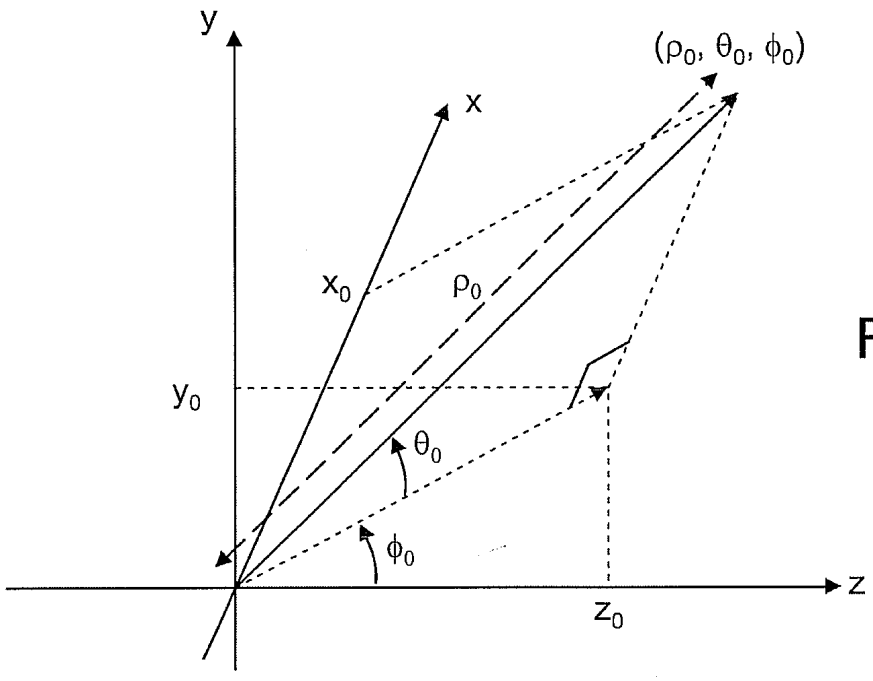
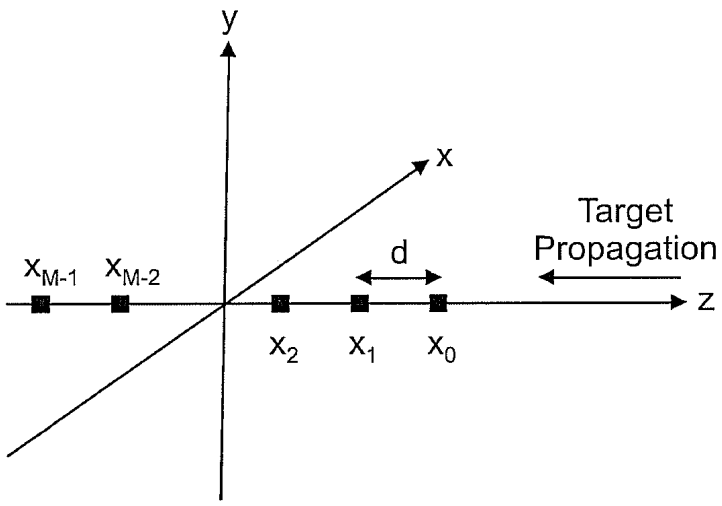
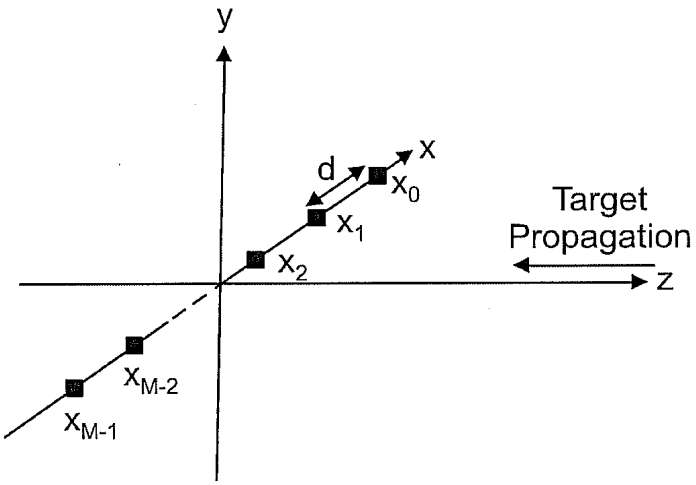

y
x
$(\rho_0, \theta_0, \phi_0)$
x_0
ρ_0
y_0
θ_0
ϕ_0
z
z_0

FIG. 2



Endfire

FIG. 3A



Broadside

FIG. 3B

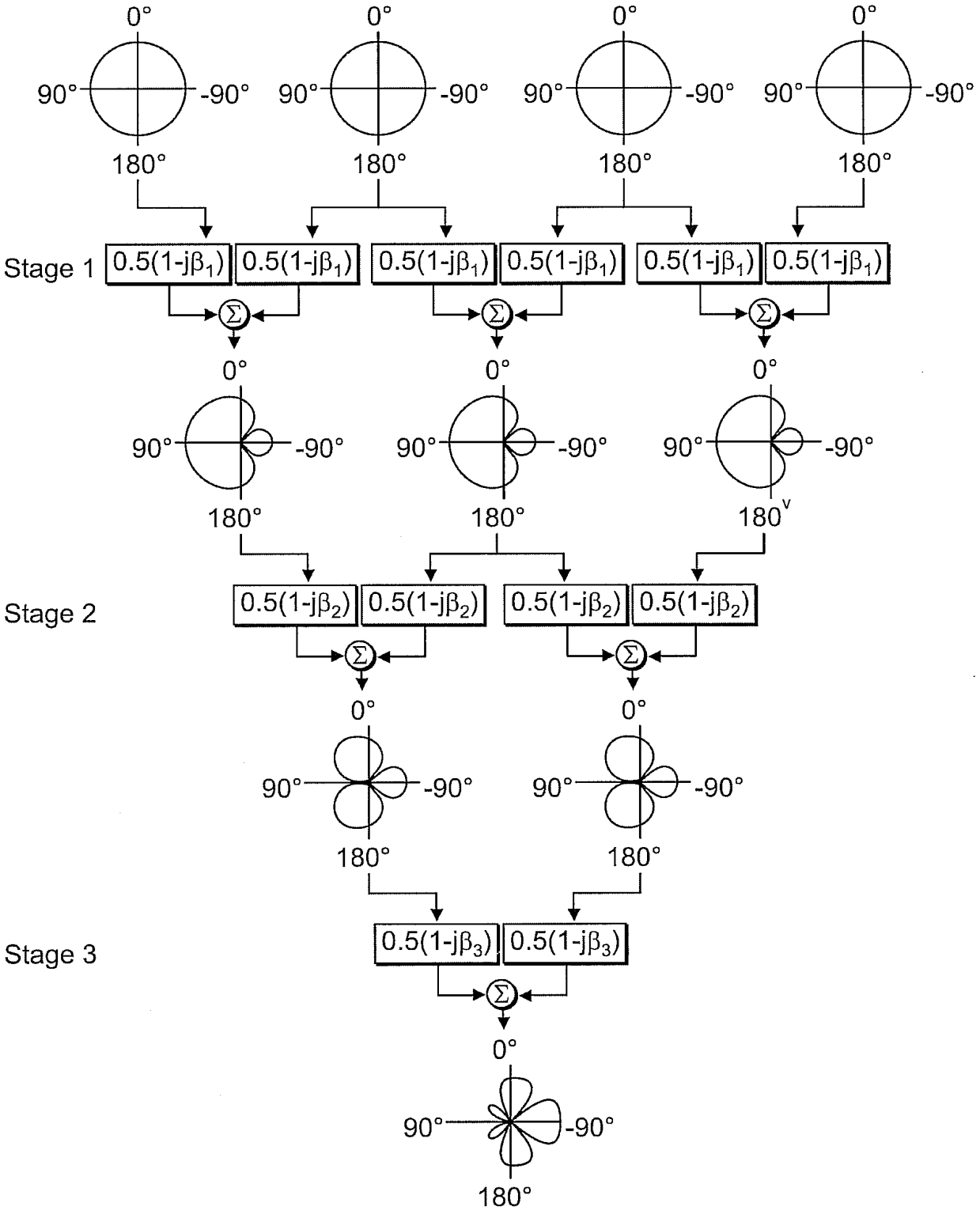


FIG. 4

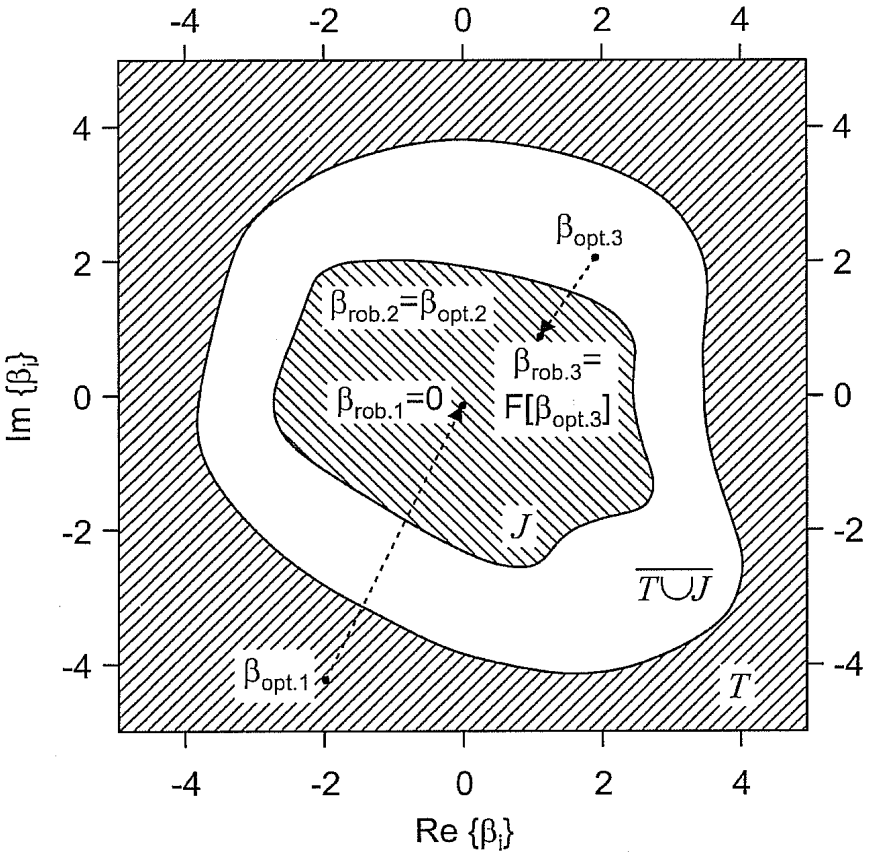

-4
-2
0
2
4
Im {βi}
Re {βi}
βopt.3
βrob.2=βopt.2
βrob.3=
F[βopt.3]
βrob.1=0
J
T∪J
βopt.1
T

FIG. 5

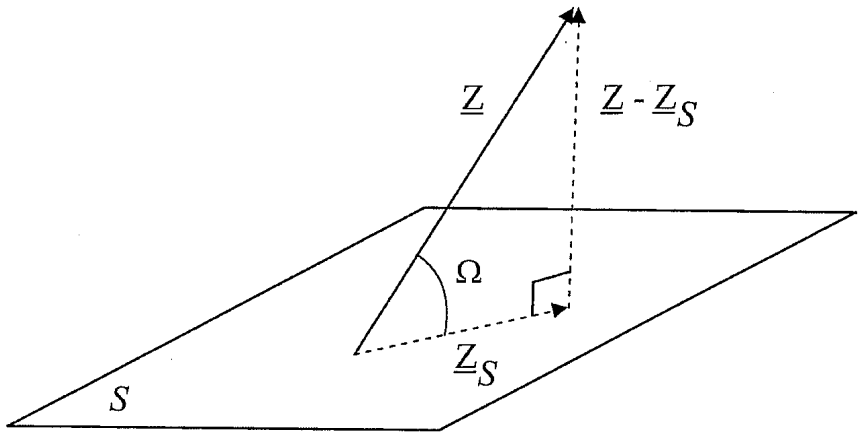
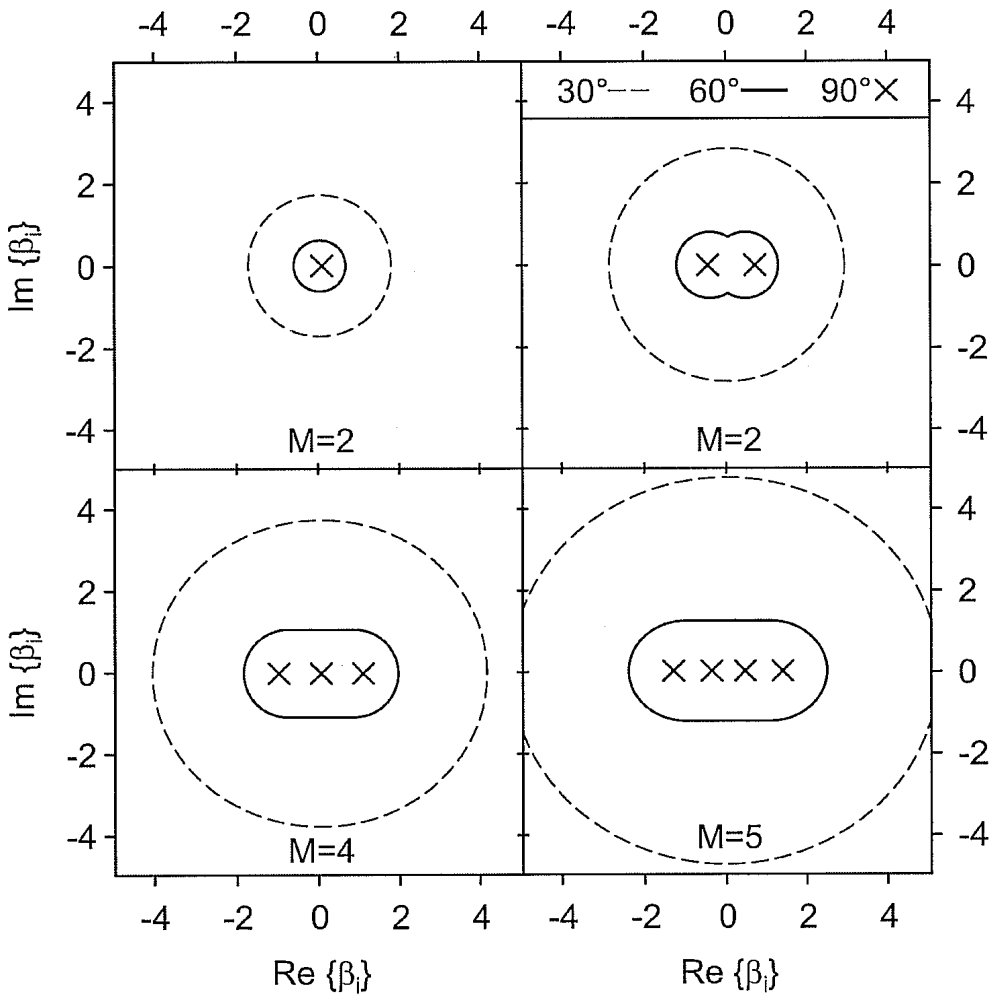

Z
Z - ZS
Ω
ZS
S

FIG. 6



Contours for $\Omega_i(\beta) = 30°$ (--) 60° (—), and 90° (×) for M = 2, 3, 4 and 5 element arrays.

FIG. 7

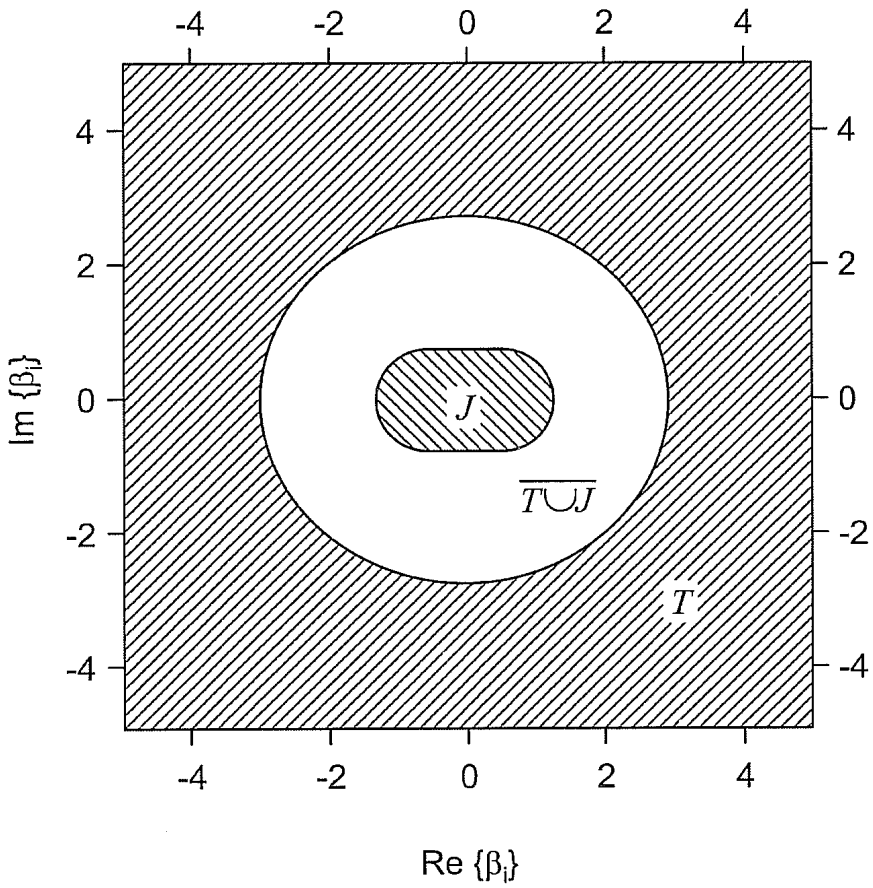
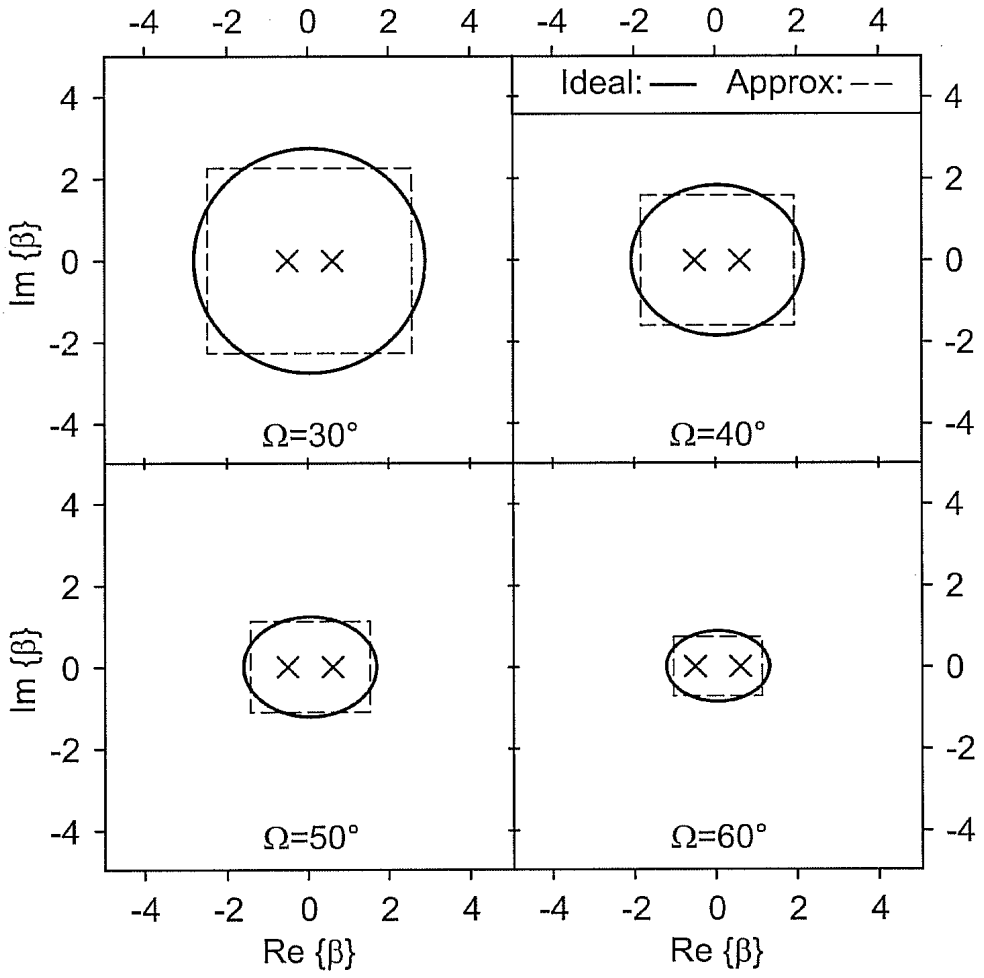


FIG. 8



Contours (——), and approximated contours (– –) for M = 3 and $\Omega(\beta_i)$ = 30°, 40°, 50°, and 60°, with β_i^+ also indicated (×).

FIG. 9

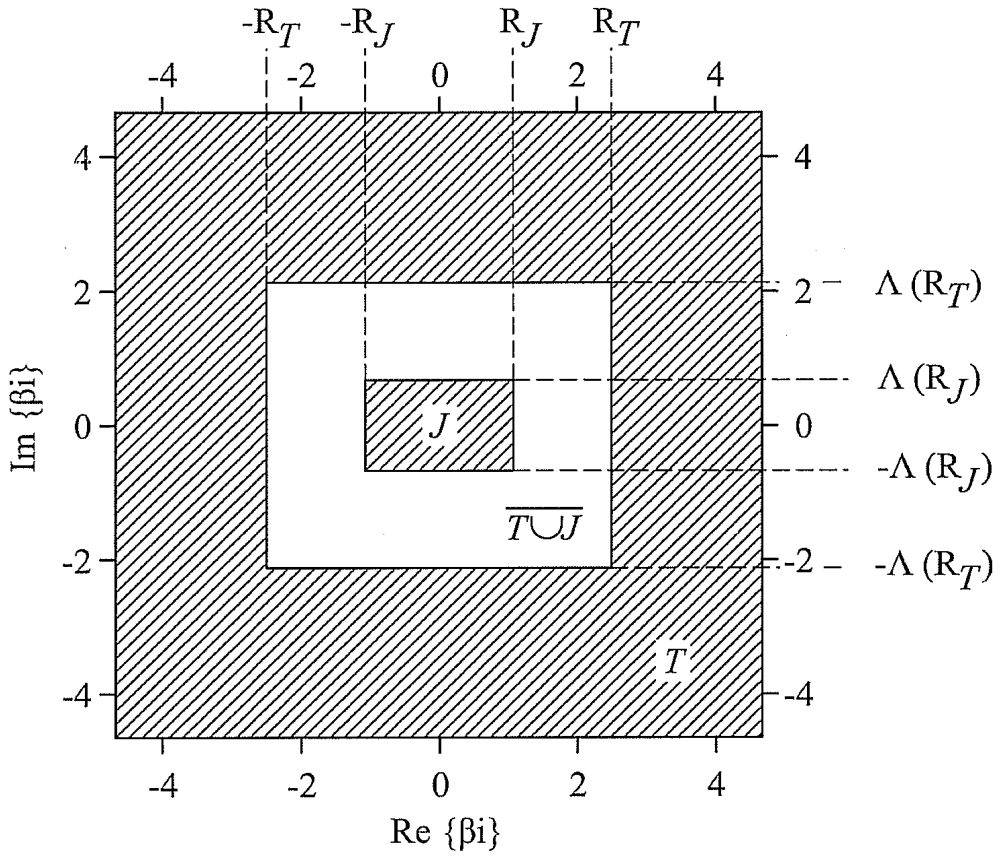
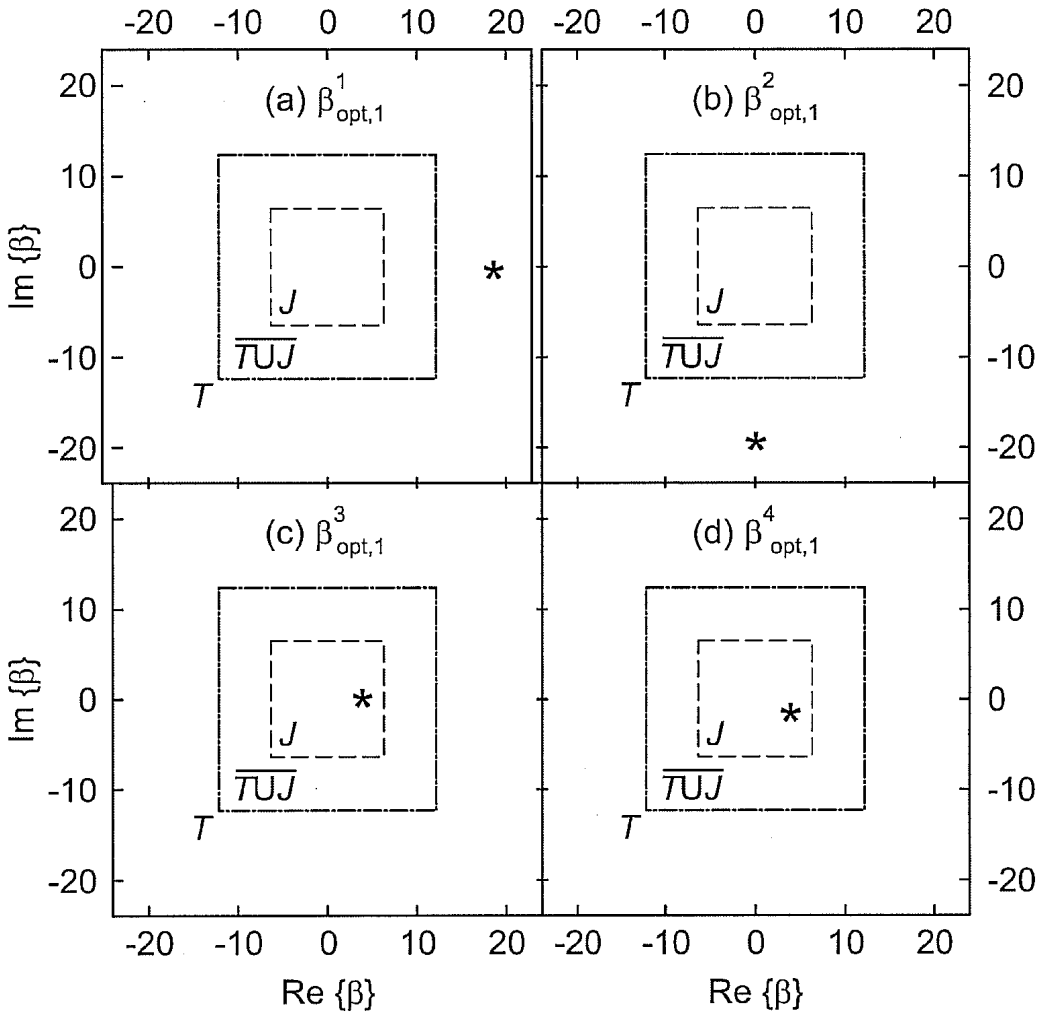


Diagram showing the three modified LENS parameter classification regions for an M=3 element array as according to Equation 3.23 when R_T and R_J are chosen to reflect the $\Omega_T = 30°$ and $\Omega_J = 60°$ contours, respectively.

FIG. 10



Four example values of $\beta_{opt,1}$ for two-element array along along with the corresponding LENS robustness regions.

FIG. 11

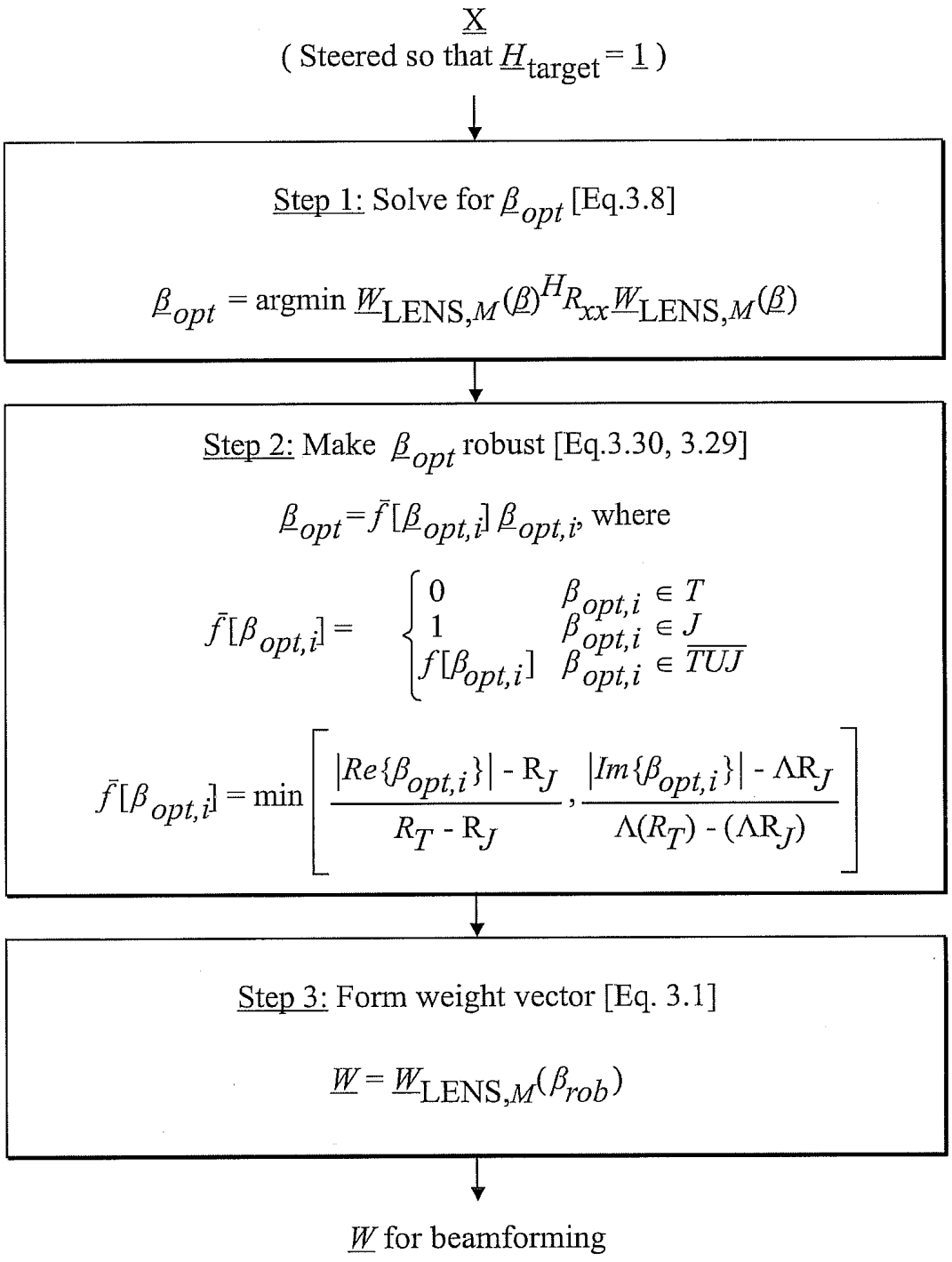

X
(Steered so that $\underline{H}_{\text{target}} = \underline{1}$)
Step 1: Solve for $\underline{\beta}_{opt}$ [Eq.3.8]
$\underline{\beta}_{opt} = \text{argmin } \underline{W}_{\text{LENS},M}(\underline{\beta})^H R_{xx} \underline{W}_{\text{LENS},M}(\underline{\beta})$
Step 2: Make $\underline{\beta}_{opt}$ robust [Eq.3.30, 3.29]
$\underline{\beta}_{opt} = \bar{f}[\underline{\beta}_{opt,i}]\, \underline{\beta}_{opt,i}$, where
$\bar{f}[\beta_{opt,i}] = \begin{cases} 0 & \beta_{opt,i} \in T \\ 1 & \beta_{opt,i} \in J \\ f[\beta_{opt,i}] & \beta_{opt,i} \in \overline{TUJ} \end{cases}$
$\bar{f}[\beta_{opt,i}] = \min \left[\frac{|Re\{\beta_{opt,i}\}| - R_J}{R_T - R_J}, \frac{|Im\{\beta_{opt,i}\}| - \Lambda R_J}{\Lambda(R_T) - (\Lambda R_J)} \right]$
Step 3: Form weight vector [Eq. 3.1]
$\underline{W} = \underline{W}_{\text{LENS},M}(\beta_{rob})$
$\underline{W}$ for beamforming

FIG. 12

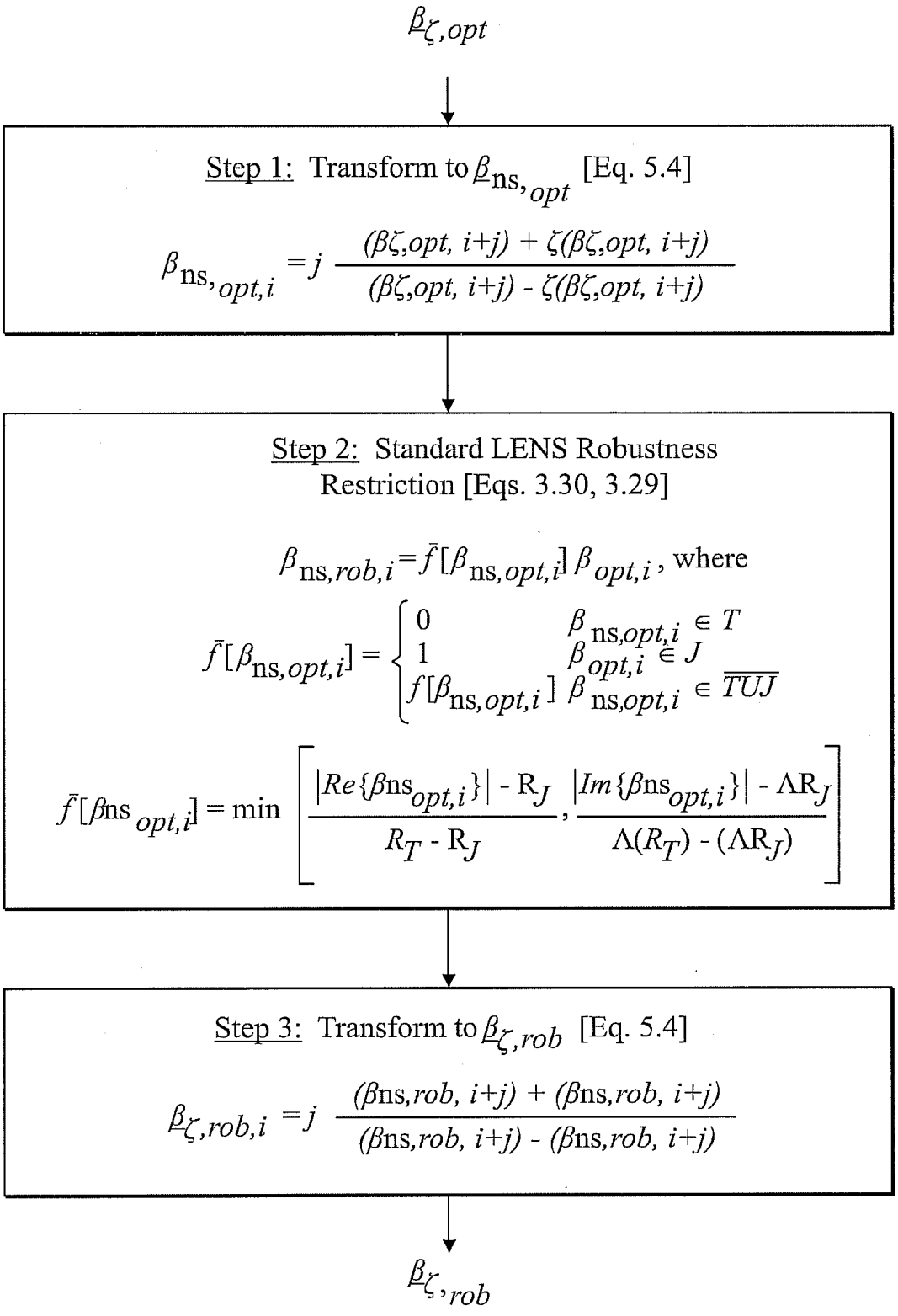

$\beta_{\zeta,opt}$
Step 1: Transform to $\beta_{ns,opt}$ [Eq. 5.4]
$\beta_{ns,opt,i} = j \frac{(\beta\zeta,opt,\ i+j) + \zeta(\beta\zeta,opt,\ i+j)}{(\beta\zeta,opt,\ i+j) - \zeta(\beta\zeta,opt,\ i+j)}$
Step 2: Standard LENS Robustness Restriction [Eqs. 3.30, 3.29]
$\beta_{ns,rob,i} = \bar{f}[\beta_{ns,opt,i}]\,\beta_{opt,i}$, where
$\bar{f}[\beta_{ns,opt,i}] = \begin{cases} 0 & \beta_{ns,opt,i} \in T \\ 1 & \beta_{opt,i} \in J \\ f[\beta_{ns,opt,i}] & \beta_{ns,opt,i} \in \overline{TUJ} \end{cases}$
$\bar{f}[\beta ns_{opt,i}] = \min \left[\frac{|Re\{\beta ns_{opt,i}\}| - R_J}{R_T - R_J}, \frac{|Im\{\beta ns_{opt,i}\}| - \Lambda R_J}{\Lambda(R_T) - (\Lambda R_J)} \right]$
Step 3: Transform to $\beta_{\zeta,rob}$ [Eq. 5.4]
$\beta_{\zeta,rob,i} = j \frac{(\beta ns,rob,\ i+j) + (\beta ns,rob,\ i+j)}{(\beta ns,rob,\ i+j) - (\beta ns,rob,\ i+j)}$
$\beta_{\zeta,rob}$

FIG. 13

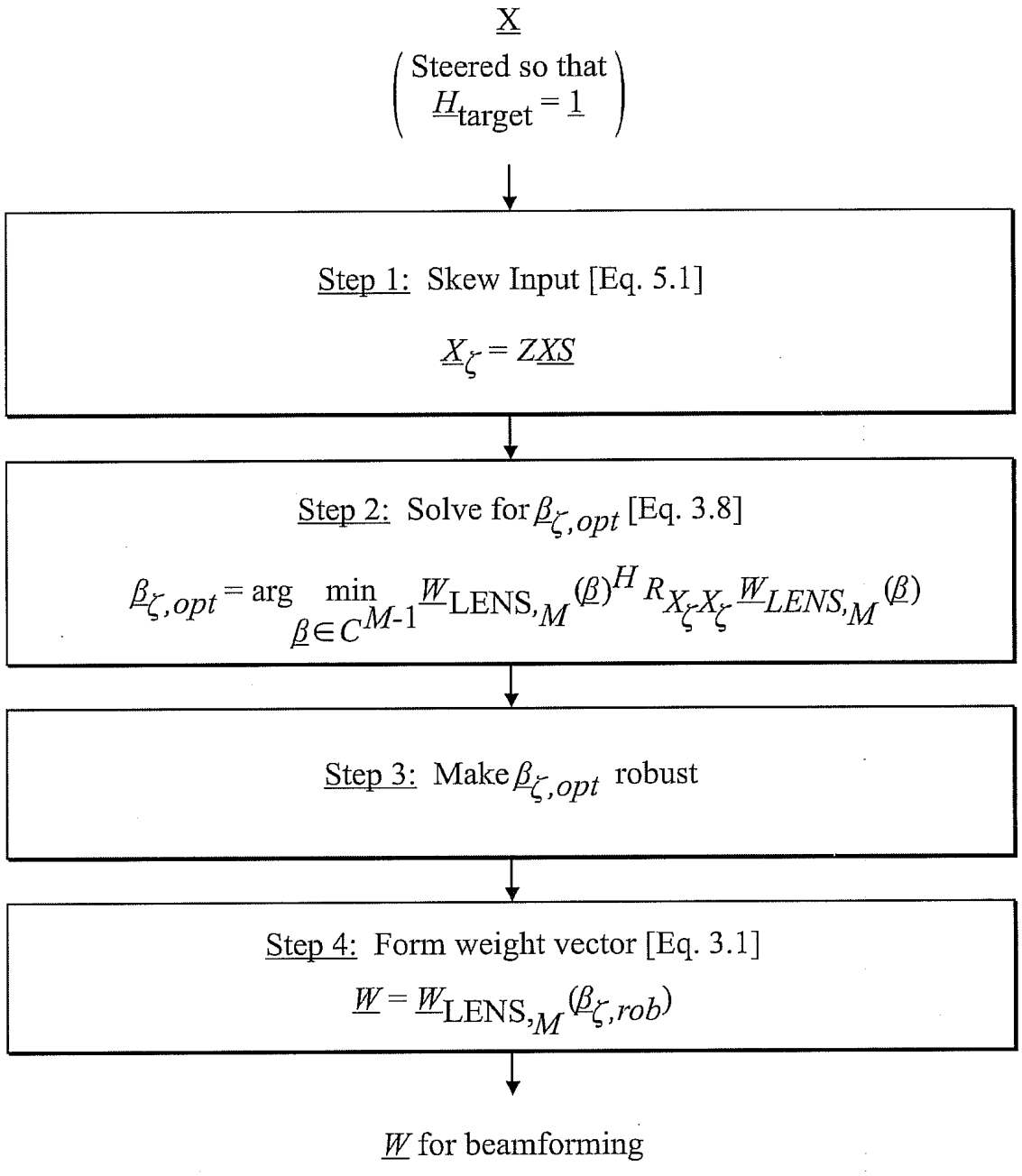

X
(Steered so that $H_{target} = 1$)
Step 1: Skew Input [Eq. 5.1]
$X_\zeta = ZXS$
Step 2: Solve for $\beta_{\zeta,opt}$ [Eq. 3.8]
$\beta_{\zeta,opt} = \arg \min_{\beta \in C^{M-1}} W_{LENS,M}(\beta)^H R_{X_\zeta X_\zeta} W_{LENS,M}(\beta)$
Step 3: Make $\beta_{\zeta,opt}$ robust
Step 4: Form weight vector [Eq. 3.1]
$W = W_{LENS,M}(\beta_{\zeta,rob})$
W for beamforming

FIG. 14

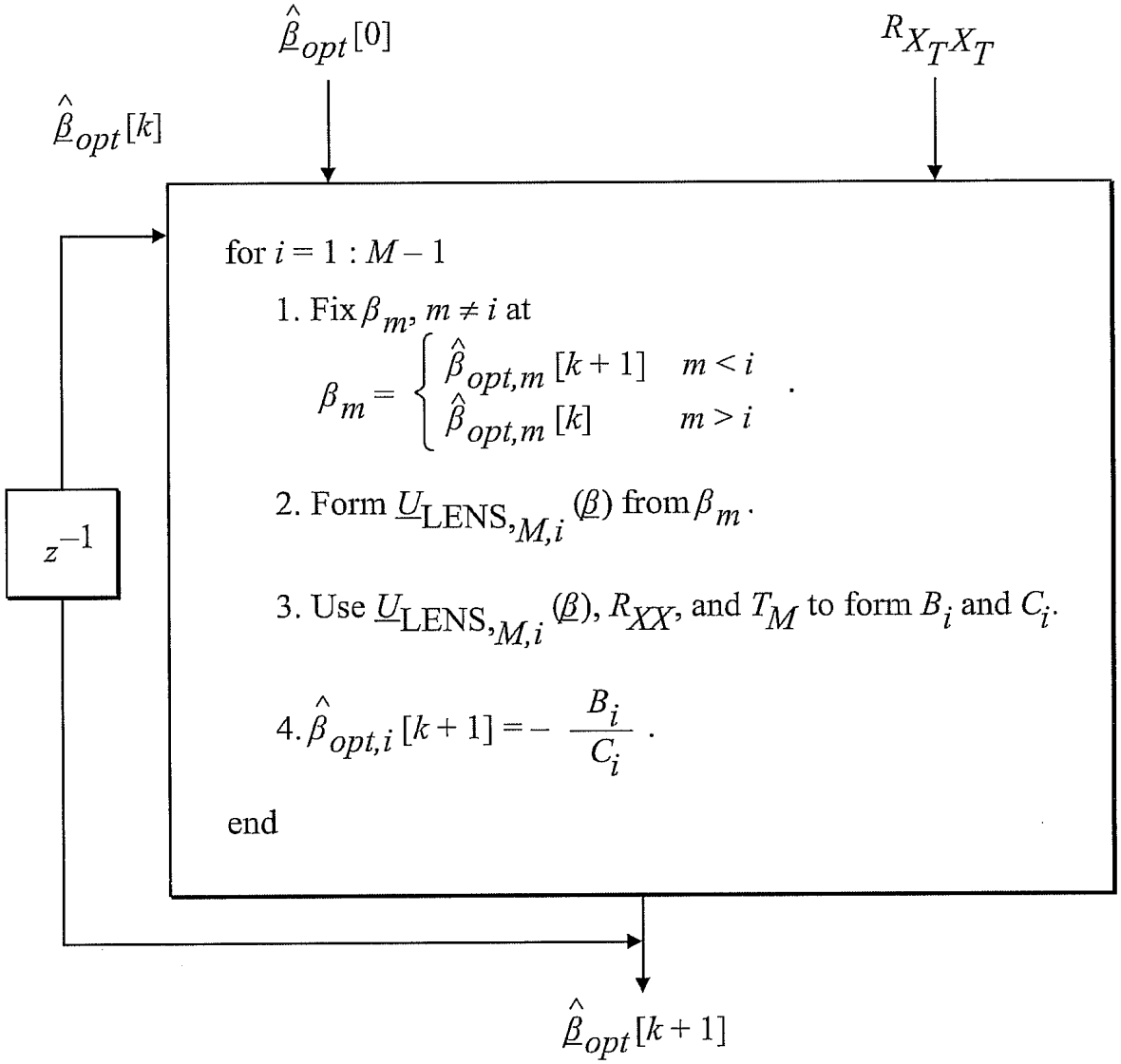
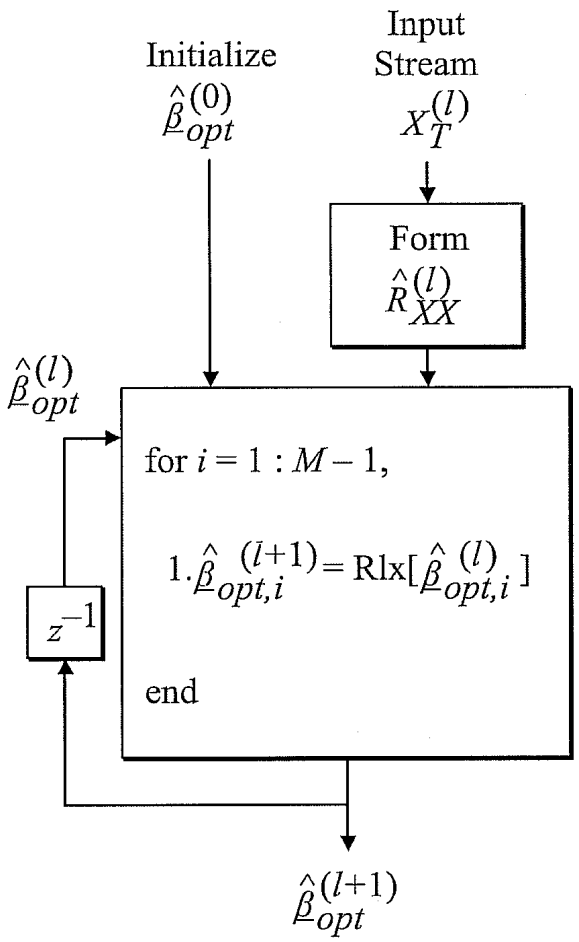
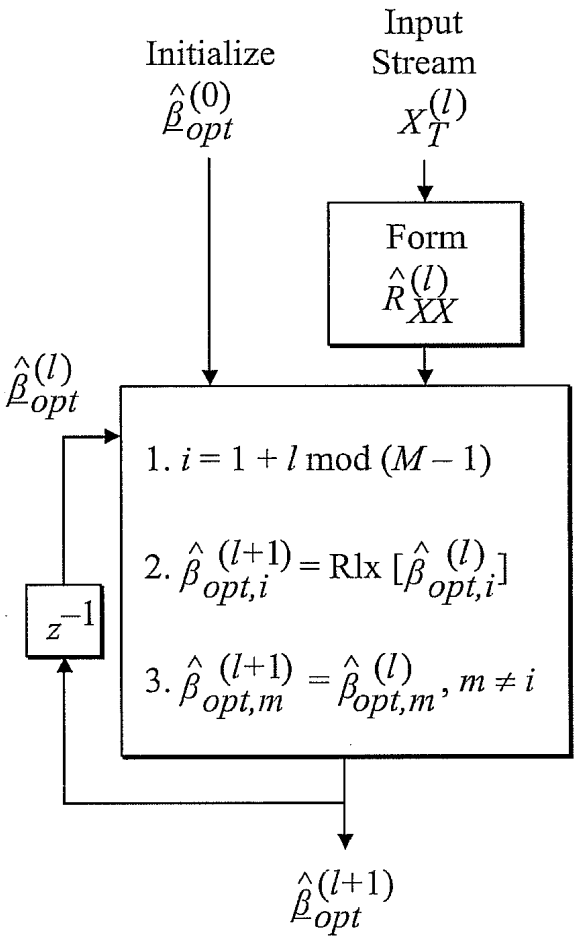

$\hat{\underline{\beta}}_{opt}[0]$
$R_{X_T X_T}$
$\hat{\underline{\beta}}_{opt}[k]$
for $i = 1 : M-1$
1. Fix β_m, $m \neq i$ at
$\beta_m = \begin{cases} \hat{\beta}_{opt,m}[k+1] & m < i \\ \hat{\beta}_{opt,m}[k] & m > i \end{cases}$.
2. Form $\underline{U}_{\mathrm{LENS},M,i}(\underline{\beta})$ from β_m.
3. Use $\underline{U}_{\mathrm{LENS},M,i}(\underline{\beta})$, R_{XX}, and T_M to form B_i and C_i.
4. $\hat{\beta}_{opt,i}[k+1] = -\frac{B_i}{C_i}$.
end
z^{-1}
$\hat{\underline{\beta}}_{opt}[k+1]$

FIG. 15



Complete Update

FIG. 16A



Partial Update

FIG. 16B

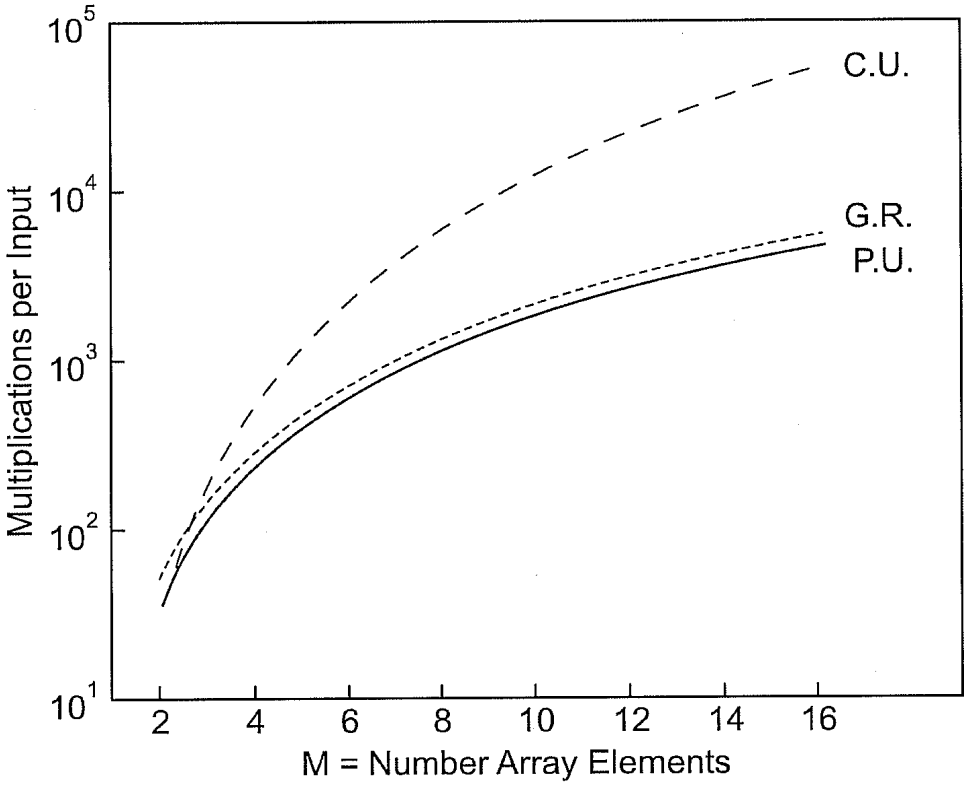

10^5
10^4
10^3
10^2
10^1
Multiplications per Input
C.U.
G.R.
P.U.
2
4
6
8
10
12
14
16
M = Number Array Elements

FIG. 17